

Centre de Physique Théorique* - CNRS - Luminy, Case 907
F-13288 Marseille Cedex 9 - France

EFFECTIVE CHARGES IN NON-ABELIAN GAUGE THEORIES[†]

N.J. Watson[‡]

Abstract

The question of the definition of effective charges for non-abelian gauge theories is discussed, focusing in particular on both the pinch technique and background field method approaches. It is argued that there does exist a unique generalization of the QED concept of an effective charge to non-abelian theories, and that this generalization is given by the pinch technique. The discussion is set in the wider (and controversial) context of the definition of gauge-independent Green function-like quantities in general in such theories.

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anonymous ftp or gopher: cpt.univ-mrs.fr

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[‡]email: watson@cptsu4.univ-mrs.fr

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N.J. WATSON

*Centre de Physique Théorique, CNRS Luminy,
F-13288 Marseille Cedex 9, France.
E-mail: watson@cptsu4.univ-mrs.fr*

ABSTRACT

The question of the definition of effective charges for non-abelian gauge theories is discussed, focusing in particular on both the pinch technique and background field method approaches. It is argued that there does exist a unique generalization of the QED concept of an effective charge to non-abelian theories, and that this generalization is given by the pinch technique. The discussion is set in the wider (and controversial) context of the definition of gauge-independent Green function-like quantities in general in such theories.

1. Introduction

With the advent of the LEP machine at CERN, the study of radiative corrections to the tree level predictions of the electroweak sector of the Standard Model (SM) has become of enormous importance. Because these corrections are formulated in the framework of perturbation theory, in which one necessarily has to break the gauge invariance of the classical lagrangian, issues of gauge dependence naturally arise. While physical observables are known from general proofs to be independent of the particular gauge fixing procedure used, the intermediate steps of any perturbative calculation involve Green functions which in general are gauge-dependent. Although this gauge dependence necessarily cancels in the end, the gauge dependence of the individual Green functions may be very strong, even introducing spurious singularities. Indeed, in some gauges the behaviour of the individual Green functions may be such as to obscure completely important characteristics of the theory. Perhaps the most spectacular example of this is the standard statement that the electroweak sector is non-renormalizable in the unitary gauge.

As long as one is only interested in calculating S-matrix elements in a given theory, this gauge dependence of the Green functions does not represent a problem. But in attempting to parameterize and detect deviations from the SM predictions resulting from the effects of “new physics”, particularly at accuracies beyond tree level, the fact that the basic building blocks of perturbation theory are gauge-dependent causes difficulties. For example, in attempting to parameterize the deviations from the SM tree level predictions for the electroweak triple gauge vertices soon to be measured at LEP, the conventional one-loop proper three-point functions calculated in the SM for off-shell particles are not only gauge-dependent, but in the Feynman gauge involve contributions which are both infrared divergent and badly behaved at high energies.

This is before any attempt has been made to include the effects of possible non-SM physics.

Over the last few years there have been various suggestions^{1–5} as to how to reorganize in a more physical and intuitive way the Green functions for one-loop radiative corrections occurring in non-abelian gauge theories. The basic idea is to rearrange parts of the contributions from the various one-loop n -point functions to form sets of gauge-independent self-energy-like, vertex-like and box-like functions. The most systematic of these schemes is the pinch technique (PT), introduced originally by Cornwall^{6–9} in the context of QCD, and since extensively developed and applied by Papavassiliou and collaborators^{10–17}. The PT provides a well-defined algorithm for the rearrangement of one-loop corrections to tree-level processes, with the resulting functions, in addition to being gauge-independent, displaying many theoretically desirable properties. In particular, the PT one-loop functions satisfy the same set of Ward identities as the corresponding tree level Green functions. Furthermore, it has been shown by Degrandi and Sirlin¹⁶ that the PT algorithm in fact corresponds to a systematic use of current algebra, thus demonstrating explicitly the PT's basis in the underlying gauge symmetry of the theory.

An alternative approach to the calculation of radiative corrections is the background field method (BFM)^{18–25}. By splitting the gauge fields into background and quantum components and then choosing a gauge fixing for the quantum fields such that explicit gauge invariance of the background fields is retained, the BFM provides a very appealing framework in which to carry out the calculation of radiative corrections. This has recently been demonstrated explicitly with the application of the BFM to the electroweak sector of the SM^{26–27}. It has been shown that the PT functions are obtained directly in the BFM as the background field Green functions for the particular choice of the Feynman quantum gauge $\xi_q = 1$ ^{28–29}, while all of the desirable properties of the PT functions are obtained in the BFM for *any* choice of the quantum gauge parameter ξ_q ^{26–27}. This includes the background field Green functions satisfying to all orders the PT tree level-like Ward identities, a direct result of the exact background gauge invariance of the BFM effective action. It was argued^{26–27} that the PT is therefore not distinguished on physical grounds, but rather represents just one of an infinity of choices to obtain well-behaved Green function-like quantities, this choice being parameterized in the BFM by the quantum gauge fixing parameter ξ_q .

If this conclusion is correct, then clearly it makes (or rather leaves) difficult the analysis and interpretation of the triple gauge vertex measurements at LEP: beyond tree level, what one regards as the vertex function must be a matter of convention. However, it will be argued here that the PT precisely *is* distinguished on physical grounds from the general BFM approach. The discussion centres on the specific question, does there exist a unique, unambiguous way to extend the concept of an effective charge from QED to non-abelian gauge theories? It will therefore concentrate

on the gauge boson two-point function. This question is of interest not just from a phenomenological point of view but also, because the effective charge sums an infinite Dyson series of radiative corrections and so goes beyond perturbation theory, is central to renormalon approaches to QCD. After reviewing the problem, a discussion is given of the gauge boson two-point functions obtained in both the PT and BFM approaches. It is then argued that there does indeed exist a natural generalization of the idea of an effective charge to non-abelian theories, this generalization being given by the PT. For simplicity, only the case of an unbroken non-abelian theory—SU(N) QCD—is considered.

2. The Gauge Boson Two-Point Function

In the classical lagrangian for QED,

$$\mathcal{L}_{\text{cl}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}[i\gamma^\mu(\partial_\mu - ieA_\mu) - m]\psi \quad (1)$$

there is only a single interaction vertex, viz. that of the gauge boson (photon) with a fermion-antifermion (electron-positron) pair. The photon self-energy due to this interaction is transverse and gauge-independent to all orders in perturbation theory:

$$A_\mu \text{ --- } q \text{ --- } \text{[diagram: a circle with diagonal lines representing a fermion loop]} \text{ --- } q \text{ --- } A_\nu = (q^2 g_{\mu\nu} - q_\mu q_\nu) i\Pi(q^2). \quad (2)$$

After renormalization, this 1PI photon self-energy may be summed in a Dyson series to give, at a given order of perturbation theory, the renormalized dressed photon propagator

$$i\Delta_{R\mu\nu}(q) = \frac{i}{q^2 + i\epsilon} \left\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) d_R(q^2) - \xi \frac{q_\mu q_\nu}{q^2} \right\} \quad (3)$$

where ξ is the gauge parameter in the class of conventional R_ξ gauges. This photon propagator then naturally defines a gauge-independent effective charge for the abelian theory:

$$e_R^2 d_R(q^2) = \frac{e_R^2}{1 - \Pi_R(q^2)} = e_{\text{eff}}^2(q^2) \quad (4)$$

where e_R is the renormalized coupling constant. At $q^2 = 0$ (the Thomson limit), this effective charge matches on to the fine structure constant ($e_{\text{eff}}^2(0)/4\pi = \alpha = 1/137.03\dots$). Furthermore, in addition to being gauge-independent, as a result of the famous QED relation $Z_1 = Z_2$, the QED effective charge is also renormalization scale-independent. At asymptotic q^2 values, it therefore obeys a homogeneous Callan-Symanzik equation involving the QED β function.

In the classical lagrangian for QCD with n_f flavours of fermion,

$$\mathcal{L}_{\text{cl}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_{f=1}^{n_f} \bar{\psi}^{(f)} [i\gamma^\mu (\partial_\mu - igA_\mu^a T^a) - m_f] \psi^{(f)} \quad (5)$$

in addition to the interaction of the gauge bosons (gluons) with the fermions (quarks) similar to that of QED, the gauge bosons also couple directly to one another in triple and quadruple gauge boson vertices. Although as in QED a Ward (more correctly Slavnov-Taylor) identity guarantees that the gauge boson self-energy is transverse, as a result of the gauge boson self-interactions it is gauge-dependent:

$$A_\mu^a \text{ --- } q \text{ --- } \text{[shaded circle]} \text{ --- } q \text{ --- } A_\nu^b = (q^2 g_{\mu\nu} - q_\mu q_\nu) \delta^{ab} i\Pi(\xi, q^2). \quad (6)$$

Furthermore, while it is possible after renormalization to sum this self-energy in a Dyson series to give a radiatively-corrected gauge field propagator, the quantity $g_{\text{eff}}(\xi, q^2)$ defined by analogy with the QED effective charge Eq. (4) does not at all have the high energy behaviour expected from the QCD β function. The simple QED correspondence between the gauge boson two-point function and an effective charge for the theory is therefore lost.

3. The Pinch Technique

The pinch technique (PT) is based on the observation⁶⁻⁹ that in a non-abelian gauge theory, one-loop diagrams which appear to give only vertex or box corrections to tree level processes in fact implicitly contain propagator-like components. It is important to emphasize immediately that this statement is not simply to do with the kinematics of a given process, but has a precise mathematical expression in terms of the tree level Feynman rules of the theory.

In order to illustrate this, consider the four fermion scattering process $\psi_i^{(f)} \psi_{i'}^{(f')} \rightarrow \psi_j^{(f)} \psi_{j'}^{(f')}$ in $\text{SU}(N)$ QCD, Eq. (5). The complete set of one-loop diagrams for this process is shown in Fig. 1. The contribution of the diagram in Fig. 1(a) involving the conventional gauge boson two-point function is given by

$$\text{Fig. 1(a)} = \left(\bar{u}_{j'} i g \gamma^\alpha T_{j'i'}^a u_{i'} \right) \frac{-i}{q^2} i q^2 \Pi(\xi, q^2) \frac{-i}{q^2} \left(\bar{u}_j i g \gamma_\alpha T_{ji}^a u_i \right) \quad (7)$$

(the particular indices i, i', j, j' are not summed). The effect of the PT algorithm is to extract the propagator-like components of the remaining diagrams in Fig. 1, *defined as those parts of the diagrams which have exactly the form of Eq. (7), i.e. a function of q^2 between two tree level vector-fermion-fermion vertices*. These are the pinch parts of the diagrams, shown in Figs. 1(c) and (e). Adding these pinch parts

of the vertex and box diagrams to the diagram Fig. 1(a) involving the conventional gauge boson two-point function gives the full one-loop propagator-like contribution to the four fermion process, illustrated schematically in Fig. 1(h), and defines the PT gauge boson self-energy $\hat{\Pi}(q^2)$:

$$\text{Fig. 1(h)} = \left(\bar{u}_{j'} i g \gamma^\alpha T_{j'i'}^a u_{i'} \right) \frac{-i}{q^2} i q^2 \hat{\Pi}(q^2) \frac{-i}{q^2} \left(\bar{u}_j i g \gamma_\alpha T_{ji}^a u_i \right). \quad (8)$$

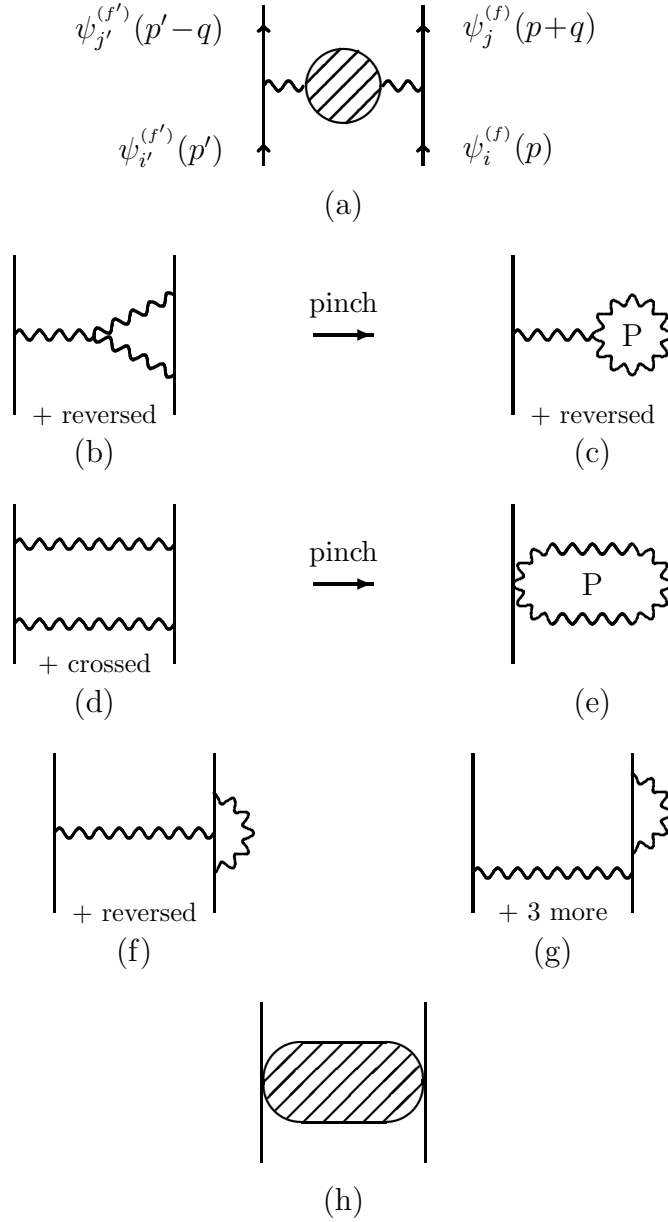


Fig. 1. The set of one-loop diagrams for the four fermion scattering process and their pinch parts, denoted by “P”. The pinch technique self-energy is shown in (h).

Having defined $\hat{\Pi}(q^2)$ in this way, one can now argue that it must be gauge-independent: up to a trivial dependence on the external spinors, the component Eq. (8) of the S-matrix element for the four fermion process depends only on the t -channel momentum transfer q^2 , and not on the s -channel momentum transfer $(p+p')^2$ or the external fermions' masses. It must therefore be individually gauge-independent, as can be verified by explicit calculation.

The identification of the pinch parts is made at the level of the Feynman integrals

for the diagrams using the tree level vector-fermion-fermion vertex Ward identity

$$q^\alpha \gamma_\alpha = S^{-1}(p+q) - S^{-1}(p) \quad (9)$$

where $S^{-1}(p) = \not{p} - m$ is the inverse fermion propagator, q is the four-momentum of the incoming gauge boson and p and $p+q$ are the four-momenta of the fermions. In general, such factors of four-momentum arise both from the longitudinal components of the gauge field propagators and from triple gauge vertices. The latter may be decomposed as

$$g\Gamma_{\alpha\beta\gamma}^{abc} = gf^{abc}(\Gamma_{\alpha\beta\gamma}^F + \Gamma_{\alpha\beta\gamma}^P) \quad (10)$$

where

$$\Gamma_{\alpha\beta\gamma}^F(q, k, -k-q) = (2k+q)_\alpha g_{\beta\gamma} - 2q_\beta g_{\gamma\alpha} + 2q_\gamma g_{\alpha\beta} \quad (11)$$

$$\Gamma_{\alpha\beta\gamma}^P(q, k, -k-q) = -k_\beta g_{\gamma\alpha} - (k+q)_\gamma g_{\alpha\beta}. \quad (12)$$

The part $\Gamma_{\alpha\beta\gamma}^F$ gives no pinch contribution and obeys a simple QED-like Ward identity involving the difference of two inverse gauge field propagators in the Feynman gauge, $q^\alpha \Gamma_{\alpha\beta\gamma}^F(q, k, -k-q) = [(k+q)^2 - k^2]g_{\beta\gamma}$. The part $\Gamma_{\alpha\beta\gamma}^P$ gives two pinch contributions, one from k_β , the other from $(k+q)_\gamma$. Use of the Ward identity Eq. (9) for *all* such factors of longitudinal four-momentum appearing in the denominator of the integral enables one to isolate the components of the diagrams in which the fermion propagators associated with the external lines are exactly cancelled.

Given that the PT gauge boson self-energy $\hat{\Pi}(q^2)$ is gauge-independent, it may be calculated using the most convenient choice of gauge. Evidently, this is the Feynman gauge $\xi = 1$, since then the gauge field propagators $iD_{\mu\nu}^{ab}$ are proportional to $g_{\mu\nu}$, leaving only the triple gauge vertex as a source of longitudinal four-momentum factors. The box diagrams Fig. 1(d) therefore have zero pinch part in this gauge, and the pinch contribution to $\hat{\Pi}(q^2)$ is given entirely by the diagrams in Fig. 1(c).

The vertex in Fig. 1(b) is shown in more detail in Fig. 2. Using the Ward identity Eq. (9), the vertex diagram in Fig. 2 can be written

$$\begin{aligned} \text{Fig. 2} = & \mu^{2\epsilon} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2(k+q)^2} \left\{ -gf^{abc} \Gamma_{\alpha\beta\gamma}^F(q, k, -k-q) ig\gamma^\gamma T^c iS(p-k) ig\gamma^\beta T^b \right. \\ & \left. + \frac{1}{2}ig^2 N \left(ig\gamma_\alpha T^a S(p-k) S^{-1}(p) + S^{-1}(p+q) S(p-k) ig\gamma_\alpha T^a \right) \right\} \\ & - ig^2 NA(q) ig\gamma_\alpha T^a \end{aligned} \quad (13)$$

where $A(q)$ is defined by

$$A(q) = \mu^{2\epsilon} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2(k+q)^2} \quad (14)$$

(we use always dimensional regularization in $n = 4 - 2\epsilon$ dimensions with 't Hooft mass scale μ). When contracted into the spinors for the on-shell external fermions,

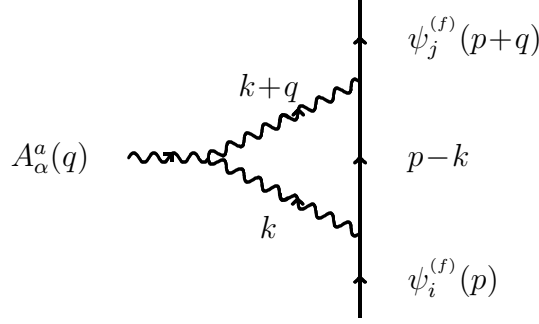


Fig. 2. The one-loop vector-fermion-fermion vertex contributing a pinch part.

the terms in the second line of Eq. (13) vanish identically. The term in the third line proportional to the tree level vector-fermion-fermion vertex $ig\gamma_\alpha T^a$ is the pinch part of the diagram. The remaining terms in the first line of Eq. (13) are part of the PT one-loop gauge-independent vector-fermion-fermion vertex, the other part being given by the “abelian-like” vertex involved in Fig. 1(f).

Given that the box diagrams give zero pinch contribution in this gauge, the PT gauge-independent gauge boson self-energy is obtained just by extracting the pinch part of the vertex in Fig. 2 together with the identical contribution from the reversed diagram, and adding them to the conventional self-energy:

$$\hat{\Pi}(q^2) = \Pi(\xi=1, q^2) - 2ig^2 N A(q). \quad (15)$$

The PT function $\hat{d}_R(q^2)$ is defined via the Dyson summation of the renormalized PT self-energy $\hat{\Pi}_R(q^2)$:

$$\hat{d}_R(q^2) = 1 + \hat{\Pi}_R(q^2) + \hat{\Pi}_R^2(q^2) + \hat{\Pi}_R^3(q^2) + \dots = \left(1 - \hat{\Pi}_R(q^2)\right)^{-1}. \quad (16)$$

Using this function, it is then possible to define the renormalized dressed gauge boson propagator

$$i\hat{\Delta}_{R\mu\nu}^{ab}(q) = \frac{i\delta^{ab}}{q^2 + i\epsilon} \left\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \hat{d}_R(q^2) - \xi \frac{q_\mu q_\nu}{q^2} \right\} \quad (17)$$

and, by analogy with Eq. (4), the corresponding PT effective charge for QCD

$$g_R^2 \hat{d}_R(q^2) = \frac{g_R^2}{1 - \hat{\Pi}_R(q^2)} = \hat{g}_{\text{eff}}^2(q^2). \quad (18)$$

At asymptotic values of q^2 , this PT effective charge satisfies a homogeneous Callan-Symanzik equation involving the QCD β function.

Thus, by extracting the parts of the one-loop vertex and box diagrams for the four fermion scattering process which are proportional to the *tree level* vector-fermion-fermion vertices, and then combining these with the conventional one-loop gauge boson self-energy, the PT self-energy $\tilde{\Pi}(q^2)$ accounts for *all* of the one-loop corrections to the single gauge boson mediating the tree level interaction between two fermion lines.

4. The Background Field Method

The background field method (BFM) provides a way of calculating the quantum corrections to tree level processes without losing explicit gauge invariance of the classical fields^{18–25}. This is achieved by first making a shift of the gauge field variable in the generating functional

$$A_\mu^a \rightarrow \tilde{A}_\mu^a + A_\mu^a \quad (19)$$

where \tilde{A}_μ^a is the background field and A_μ^a is the quantum field, the latter being the integration variable. The conventional gauge fixing term \mathcal{L}_{gf} is then replaced by two gauge fixing terms, one for the quantum fields, \mathcal{L}_{qgf} , and one for the background fields, \mathcal{L}_{bgf} :

$$-\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2 \rightarrow -\frac{1}{2\xi_q}(\tilde{D}_\mu^{ab} A^{b\mu})^2 - \frac{1}{2\xi_b}(\partial_\mu \tilde{A}^{a\mu})^2 \quad (20)$$

where \tilde{D}_μ is the background covariant derivative and an R_ξ -like gauge has been chosen for the background fields. The gauge parameters ξ_q and ξ_b are completely independent of one another. The ghost term is constructed from the variation of $\tilde{D}_\mu^{ab} A^{b\mu}$ under a quantum gauge transformation

$$\delta_q \tilde{A}_\mu^a = 0, \quad \delta_q A_\mu^a = D_\mu^{ab} \omega^b. \quad (21)$$

The terms $\mathcal{L}_{\text{cl}} + \mathcal{L}_{\text{qgf}} + \mathcal{L}_{\text{gh}}$ involved in the path integral over the quantum fields therefore remain exactly invariant under a background gauge transformation

$$\delta_b \tilde{A}_\mu^a = \tilde{D}_\mu^{ab} \omega^b, \quad \delta_b A_\mu^a = g f^{abc} A_\mu^b \omega^c. \quad (22)$$

Only the term \mathcal{L}_{bgf} , which is not involved in the path integral, breaks this exact background gauge invariance. It is important to note the distinction between background gauge invariance and quantum gauge independence: the first does not imply the second.

With a source term $J_\mu^a A^{a\mu}$ for the quantum fields added to the lagrangian, the BFM generating functional \tilde{Z} is a functional of the background fields and the sources. The generating functional $\tilde{\Gamma}[\tilde{A}_\mu^a, \langle A_\mu^a \rangle]$ for 1PI background field Green functions is given by the Legendre transform of the effective action $\tilde{W}[\tilde{A}_\mu^a, J_\mu^a] = -i \log \tilde{Z}[\tilde{A}_\mu^a, J_\mu^a]$ with respect to the sources J_μ^a . Demanding that the shift Eq. (19) be made such that

the quantum fields have zero vacuum expectation value, $\tilde{\Gamma}[\tilde{A}_\mu^a, 0]$ is exactly invariant under ordinary gauge transformations of the background fields. The background fields are then the classical fields which do not appear inside loops (since the path integral is only over the quantum fields), while the quantum fields do not appear as external particles (since $\langle A_\mu^a \rangle = 0$).

As a result of the exact invariance of $\tilde{\Gamma}$ under background gauge transformations, $\delta_b \tilde{\Gamma} = 0$, the background gauge boson self-energy is transverse:

$$\tilde{A}_\mu^a \text{ --- } q \text{ --- } \text{ (loop) } \text{ --- } q \text{ --- } \tilde{A}_\nu^b = (q^2 g_{\mu\nu} - q_\mu q_\nu) \delta^{ab} i \tilde{\Pi}(\xi_q, q^2). \quad (23)$$

However, as indicated explicitly, $\tilde{\Pi}$ retains a dependence on the quantum gauge fixing parameter ξ_q . Summing the renormalized form of this self-energy in a Dyson series gives, at a given order in perturbation theory, the dressed, renormalized background gauge field propagator

$$i \tilde{\Delta}_{R\mu\nu}^{ab}(q) = \frac{i \delta^{ab}}{q^2 + i\epsilon} \left\{ \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2} \right) \tilde{d}_R(\xi_q, q^2) - \xi_b \frac{q_\mu q_\nu}{q^2} \right\}. \quad (24)$$

It is then possible to define a BFM effective charge by analogy with Eq. (4)

$$g_R^2 \tilde{d}_R(\xi_q, q^2) = \frac{g_R^2}{1 - \tilde{\Pi}_R(\xi_q, q^2)} = \tilde{g}_{\text{eff}}^2(\xi_q, q^2). \quad (25)$$

Although the one-loop background field Green functions are in general quantum gauge-dependent, their divergent parts, and hence the renormalization counterterms, are ξ_q -independent (Kallosh's Theorem³⁰). Also, just as in QED, $\tilde{g}_{\text{eff}}(\xi_q, q^2)$ is scale-independent as a result of the BFM Ward identities. The BFM one-loop effective charge therefore has the high energy behaviour expected from the QCD β function for all values of ξ_q .

As a result of the explicitly-retained background gauge invariance of the BFM generating functionals, the background field Green functions in general obey to all orders and for all values of ξ_q the same set of Ward identities as the corresponding tree level functions. Indeed, explicit background gauge invariance is the fundamental reason for the BFM Green functions displaying for all ξ_q the theoretically desirable properties possessed by PT functions. Furthermore, the one-loop PT functions exactly coincide with the background field Green functions calculated in the particular quantum gauge $\xi_q = 1$, both in QCD²⁹ and the electroweak sector of the SM^{26–28}. Thus, for the particular case of the one-loop two-point function

$$\tilde{\Pi}(\xi_q = 1, q^2) = \hat{\Pi}(q^2) \Rightarrow \tilde{g}_{\text{eff}}^2(\xi_q = 1, q^2) = \hat{g}_{\text{eff}}^2(q^2). \quad (26)$$

On the basis of these observations, it has been argued^{26–27} that there is nothing unique about the PT functions, the PT being just one of an infinite choice of prescriptions to obtain well-behaved Green functions. In particular, this implies that, away from the asymptotic region governed by the β function, there is no unique way to define an effective charge in a non-abelian gauge theory.

5. The “Effective” Gauge Boson Two-Point Function

We will now argue against this conclusion. This involves comparing again the cases of QED and QCD, and introducing the idea of the “effective” gauge boson two-point function.

The interaction term in the QED classical lagrangian is given by

$$\mathcal{L}_{\text{cl}}^{\text{int}} = e J_\mu A^\mu \quad (27)$$

where J_μ is the electromagnetic current. At tree level, the interaction between currents at spatial points x_1 and x_2 is mediated by a single photon and is given by

$$x_1 \left| \begin{array}{c} \text{---} \text{wavy line} \text{---} \\ q \end{array} \right| x_2 = ie J^\mu(x_1) iD_{\mu\nu}(x_1 - x_2) ie J^\nu(x_2) \quad (28)$$

where the Feynman diagram in Eq. (28) is in position space and $iD_{\mu\nu}(x_1 - x_2)$ is the Fourier transform of the tree level photon propagator $iD_{\mu\nu}(q)$. Beyond tree level in perturbation theory, the renormalized interaction between the two currents at x_1 and x_2 is given by

$$x_1 \left| \begin{array}{c} \text{---} \text{wavy line} \text{---} \text{blob} \text{---} \text{wavy line \text{---} } \\ q \qquad \qquad \qquad q \end{array} \right| x_2 = ie_R J^\mu(x_1) i\Delta_{R\mu\nu}(x_1 - x_2) ie_R J^\nu(x_2) \quad (29)$$

where $i\Delta_{R\mu\nu}(x_1 - x_2)$ is the Fourier transform of the renormalized dressed photon propagator Eq. (3), involving the Dyson sum of the 1PI photon self-energy $\Pi(q^2)$. Thus, in QED the photon propagator can just as well be defined in terms of the two-point interaction between physical currents $J_\mu(x_1)$, $J_\nu(x_2)$ appearing in $\mathcal{L}_{\text{cl}}^{\text{int}}$ as from the conventional two-point Green function $\langle 0|T(A_\mu(x_1)A_\nu(x_2))|0\rangle$. Precisely because of this, the radiative corrections to the tree level propagator $iD_{\mu\nu}$ included in the dressed propagator $i\Delta_{R\mu\nu}$ can be fully accounted for essentially just by appropriately changing the coupling of the theory appearing in $\mathcal{L}_{\text{cl}}^{\text{int}}$ Eq. (27), i.e. by making the replacement $e \rightarrow e_{\text{eff}}(q^2)$, and then using the tree level propagator $iD_{\mu\nu}$.

In QCD, the interaction part of the classical lagrangian may be written

$$\mathcal{L}_{\text{cl}}^{\text{int}} = g \left(J_\mu^a + V_\mu^a + g T_\mu^a \right) A^{a\mu} \quad (30)$$

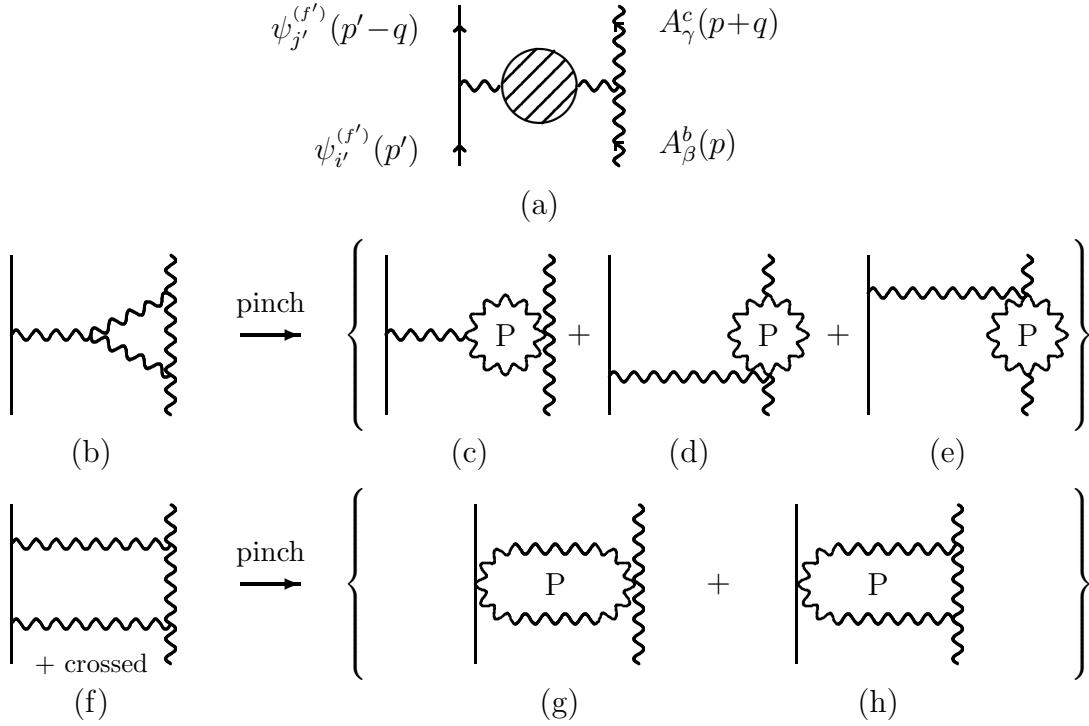


Fig. 3. The set of one-loop diagrams for the fermion-gluon scattering process (corrections involving the fermion line are not shown) and their pinch parts, denoted by “P”.

source of the difficulties described in Sec. 2. If the concept of an effective charge for the theory is to be valid, then it must account for the radiative corrections to the two-point interaction between *any* pair among the terms $J_\mu^{(f)a}$, V_μ^a , gT_μ^a in $\mathcal{L}_{\text{cl}}^{\text{int}}$. In other words, the PT self-energy must be universal.

To illustrate that this is indeed the case, consider the scattering process $A_\beta^b \psi_{i'}^{(f')} \rightarrow A_\gamma^c \psi_{j'}^{(f')}$. The set of one-loop diagrams for this process, except for those involving corrections to the fermion line, are shown in Fig. 3, together with the associated pinch diagrams. In an exactly similar way to the four fermion case in Sec. 3, the PT self-energy, shown in Fig. 3(o), is defined as the function of q^2 appearing between the tree level vertices $ig\gamma_\alpha T^a$ and $g\Gamma_{\alpha\beta\gamma}^{abc}(q, p, -p-q)$:

$$\text{Fig. 3(o)} = \left(\bar{u}_{j'} i g \gamma^\alpha T_{j'i'}^a u_{i'} \right) \frac{-i}{q^2} i q^2 \hat{\Pi}(q^2) \frac{-i}{q^2} \left(g \Gamma_{\alpha\beta\gamma}^{abc}(q, p, -p-q) \epsilon^\beta(p) \epsilon^{*\gamma}(p+q) \right) \quad (36)$$

where the ϵ 's are the external gauge boson polarization vectors.

In the diagrams shown in Fig. 3, the pinch parts are identified using the tree-level Ward identity

$$q_1^\alpha \Gamma_{\alpha\beta\gamma}(q_1, q_2, q_3) = P_{\beta\sigma}(q_3) D_{\sigma\gamma}^{-1}(q_3) - P_{\beta\sigma}(q_2) D_{\sigma\gamma}^{-1}(q_2) \quad (37)$$

where $P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$ is the transverse projection operator and $i D_{\mu\nu}(q)$ is the tree level gauge boson propagator. We work again in the Feynman gauge $\xi = 1$.

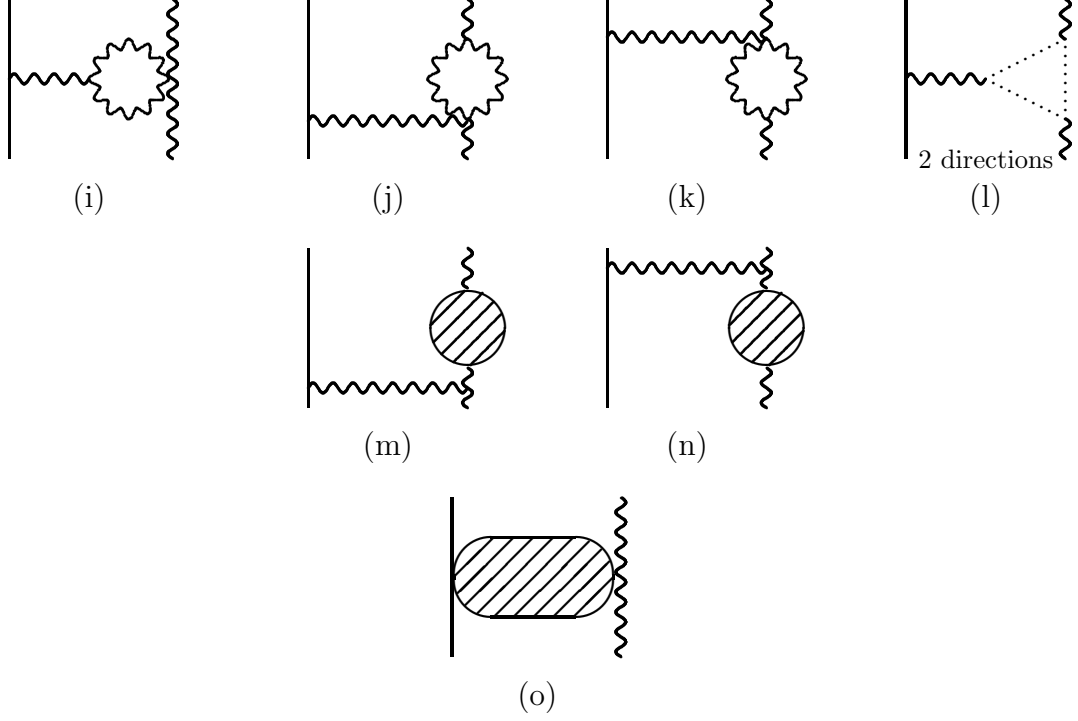


Fig. 3 continued. The pinch technique self-energy is shown in (o).

The vertex part of the diagram Fig. 3(b) is shown in more detail in Fig. 4. Using the Ward identity Eq. (37), it may be written

$$\begin{aligned}
 \text{Fig. 4} = & -\frac{ig^3 N f^{abc}}{2} \mu^{2\epsilon} \int \frac{d^n k}{(2\pi)^n} \frac{1}{k_1^2 k_2^2 k_3^2} \times \\
 & \left\{ \Gamma_{\alpha\tau\sigma}^F(q_1, k_3, -k_2) \Gamma_{\beta\rho\tau}^F(q_2, k_1, -k_3) \Gamma_{\gamma\sigma\rho}^F(q_3, k_2, -k_1) \right. \\
 & - 2(k_2 + k_3)_\alpha (k_3 + k_1)_\beta (k_1 + k_2)_\gamma + k_{2\alpha} k_{3\beta} k_{1\gamma} + k_{3\alpha} k_{1\beta} k_{2\gamma} \\
 & + q_1^2 P_{\alpha\rho}(q_1) B_{\rho\beta\gamma}(k_1, q_2, q_3) + q_2^2 P_{\beta\rho}(q_2) B_{\rho\gamma\alpha}(k_2, q_3, q_1) \\
 & \left. + q_3^2 P_{\gamma\rho}(q_3) B_{\rho\alpha\beta}(k_3, q_1, q_2) \right\} \\
 & - ig^2 N A(q_1) \left(g \Gamma_{\alpha\beta\gamma}^{abc}(q_1, q_2, q_3) + \frac{7}{4} g f^{abc} (q_{1\beta} g_{\gamma\alpha} - q_{1\gamma} g_{\beta\alpha}) \right) \\
 & - ig^2 N A(q_2) \left(g \Gamma_{\alpha\beta\gamma}^{abc}(q_1, q_2, q_3) + \frac{7}{4} g f^{abc} (q_{2\gamma} g_{\alpha\beta} - q_{2\alpha} g_{\beta\gamma}) \right) \\
 & - ig^2 N A(q_3) \left(g \Gamma_{\alpha\beta\gamma}^{abc}(q_1, q_2, q_3) + \frac{7}{4} g f^{abc} (q_{3\alpha} g_{\beta\gamma} - q_{3\beta} g_{\gamma\alpha}) \right) \quad (38)
 \end{aligned}$$

where A is given in Eq. (14) and B is given by

$$\begin{aligned}
 B_{\rho\beta\gamma}(k_1, q_2, q_3) = & -\Gamma_{\rho\beta\gamma}(q_1, q_2, q_3) - \Gamma_{\rho\beta\gamma}(0, -k_1, k_1) \\
 & - \frac{1}{2} (2k_1 + q_2 - q_3)_\rho g_{\beta\gamma} - (2k_1 + q_2)_\beta g_{\gamma\rho} - (2k_1 - q_3)_\gamma g_{\beta\rho}. \quad (39)
 \end{aligned}$$

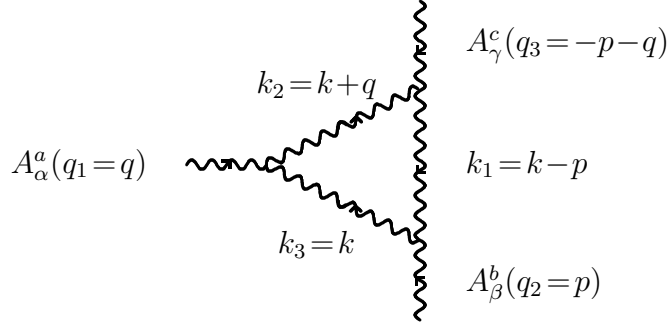


Fig. 4. The one-loop triple gauge vertex contributing pinch parts.

The three terms proportional to $g\Gamma_{\alpha\beta\gamma}^{abc}(q_1, q_2, q_3)$ in Eq. (38) are the three pinch parts of the diagram, shown in the overall amplitude in Figs. 3(c)-(e). We see immediately that each of these pinch terms has *exactly* the same dependence $-ig^2NA(q_i)$ on the corresponding momentum q_i as the pinch term proportional to $ig\gamma_\alpha T^a$ in the one-loop vector-fermion-fermion vertex Eq. (13) ($q_1 \equiv q$). Thus the pinch parts of the one-loop triple gauge vertex in Fig. 4 make exactly the required contribution, not only to the conventional self-energy appearing in the propagator Fig. 3(a) but also to that appearing in each of the external leg corrections Figs. 3(m) and (n), to give the PT self-energy $\hat{\Pi}$ (recall that there is a symmetry factor $\frac{1}{2}$ associated with the latter diagrams). Of the three terms proportional to the transverse projection operators P , those depending on $q_2 \equiv p$ and $q_3 \equiv -p - q$ vanish for the on-shell external gauge bosons. The remaining one depending on $q_1 \equiv q$ is *exactly* cancelled in the overall amplitude by the vertex-like pinch part Fig. 3(h) of box diagrams. The remaining terms in Eq. (38) are part of the PT one-loop gauge-independent triple gauge vertex, the other parts being given by the vertices in diagrams Figs. 3(i)-(k) involving the quadruple gauge vertex and in Fig. 3(l) involving the ghosts.

It is important to note that, although the loop integral for the diagram in Fig. 3(i) depends only on the four-momentum transfer q , it does *not* contribute to the PT self-energy since it is not proportional to the tree level triple gauge vertex $g\Gamma_{\alpha\beta\gamma}^{abc}$:

$$\text{Vertex of Fig. 3(i)} = -ig^2NA(q_1)\frac{9}{4}gf^{abc}(q_{1\beta}g_{\gamma\alpha} - q_{1\gamma}g_{\beta\alpha}). \quad (40)$$

Similarly, the ghost diagrams contribute only to the PT one-loop vertex. Of the remaining diagrams in Fig. 3, the propagator-like pinch part Fig. 3(g) of the box diagrams vanishes in the Feynman gauge, just as in the four fermion scattering process.

Thus, we see that exactly the same PT self-energy is obtained from the fermion-gluon scattering process as from fermion-fermion process. It has recently been shown³¹ how this result generalizes to all the combinations of fields (including scalars) to which

a gauge boson boson couples at tree level.

6. Conclusions

I have tried to argue here how the concept of an effective charge can be naturally extended from QED, where it arises automatically, to non-abelian gauge theories. The argument starts from the simple fact that, in both abelian and non-abelian theories, the charge g appearing in the interaction term $gJ \cdot A$ of the classical lagrangian defines the strength of the tree level interaction between two fermionic currents J at points x_1 and x_2 due to the exchange of a single gauge boson A . The role of the effective charge g_{eff} is then simply to account for the change in strength of this two-point interaction due to radiative corrections. In QED, this can be calculated in perturbation theory by considering just the conventional photon two-point function. But in a non-abelian theory, there are contributions to the interaction between currents at two points which are not included in the conventional non-abelian gauge boson two-point function. This is precisely the observation upon which the pinch technique of Cornwall and Papavassiliou is based; it is made particularly transparent in the current algebra formulation of the pinch technique due to Degrassi and Sirlin. Thus, to calculate the change in strength of the interaction between currents at two points due to radiative corrections, rather than considering the conventional gauge boson two-point function, one needs to consider the PT “effective” gauge boson two-point function. This then gives the effective charge for the non-abelian theory.

However, in a non-abelian theory there are further sets of fields in the interaction part of the classical lagrangian which interact with one another at tree level via the exchange of a single gauge boson with strength governed by g . If the concept of the PT “effective” gauge boson two-point function is to be valid, then it must also account for the radiative corrections between any pair of these combinations of fields at two points. The natural way in which this happens has been demonstrated for the case of the triple gauge vertex here and in general elsewhere³¹.

Returning to the general discussion of the definition of gauge-independent Green function-like quantities, the lesson to be learned from the background field method is that it is easy to define sets of well-behaved Green functions. Although the BFM Green functions themselves are certainly not gauge-independent, it is precisely *because* they are dependent on an arbitrary parameter (ξ_q) and yet still possess all of the other properties one desires that they demonstrate that some further field-theoretic criterion is required to define gauge-independent Green functions unambiguously. This I believe will turn out to be supplied by the concept of the “effective” n -point function

obtained in the pinch technique and described above for $n = 2$.

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